Polarization of Cosmic-Ray μ Mesons*

HALE V. BRADT AND GEORGE W. CLARK
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 20 May 1963)

This paper reports: (1) measurements of the decay asymmetry of positive cosmic-ray muons which come to rest in a brass absorber at sea level under 12, 200, 700 and 3700 g/cm², air equivalent, of material; and (2) a determination of limits on the possible change with energy of the kaon-pion production ratio in nuclear interactions at energies from about 10 BeV to about 100 BeV. The measurements give values of the nominal polarization equal to 0.23 ± 0.03, 0.16 ± 0.02, 0.23 ± 0.03, and 0.21 ± 0.02 in order of increasing depth. The actual polarization is somewhat higher because of unevaluated systematic errors (e.g., electron scattering) which depend upon the details of construction of a given detector. The ratios of the polarizations observed at the different depths were compared with those computed under the assumption that the relative production rate of $K_{\mu 2}^+$ and π^+ mesons per unit energy interval varies with energy according to the relation $K(E)/\pi(E) \propto (0.3+E)^\delta$, where E is the total energy measured in BeV and $\pi(E) \propto (0.3+E)^{-2.64}$. We find that the data can be fitted to this relation for any value of δ between $-\infty$ and $+0.6$. This result is rather insensitive to changes in the constants 0.3 and 2.64 and is based upon the assumptions that: (1) there is a negligible anomalous depolarization of the muons as they traverse the material above the detectors, and (2) the muons arise only from the decays of pions and charged kaons. If charged and neutral kaons are produced in equal numbers, the upper limit for δ would be raised to about 1.3.

I. INTRODUCTION

A PREVIOUS experiment at this laboratory[1] demonstrated that positive cosmic ray muons are polarized with a polarization value of 0.19 ± 0.06 for those which stop in a thin absorber under 200 g/cm² of shielding at sea level. We have now extended this work with greatly increased statistical accuracy and report here our measurements of the polarization of positive cosmic-ray muons at sea level under 12, 200, 700 and 3700 g/cm², air equivalent, of shielding. Preliminary results have been reported previously.[2–4] Since the polarization depends in part upon the relative numbers of kaons and pions among the parents of the muons, these measurements provide information on the variation of the production ratio for kaons and pions over a range of interaction energies from about 10 BeV to about 100 BeV.

The ratio of the production frequency of kaons and pions at high energies has been studied by a variety of methods. Measurements of the positive-charge excess of cosmic-ray muons are consistent with the assumption that the muons arise predominantly from pions,[5,6] although the interpretation of the data depends on the model adopted to describe the high-energy nucleonic cascade. The momentum spectra of muons at various zenith angles[7–10] indicate that most muons below 100 BeV arise from pions, but the statistical accuracy of the data presently available does not permit an accurate determination of the kaon-pion production ratio. Cloud chamber[11] and emulsion data[12–14] are based on the detailed study of individual nuclear interactions. At high energies the identification of the secondary particles is uncertain, and the number of available events is not large. The Chicago group[14] estimates, on the basis of their work and that of the Bristol group,[13] that $(30\pm6)\%$ of all particles produced in interactions at several thousand BeV are not pions. This result is consistent with the results reported by other investigators. Recently, the Bristol group[15] has studied the gamma-ray flux at high altitudes with emulsion techniques and compared this with the observed muon flux at sea level. Their results indicate that the integral production spectrum of pions is greater than that of kaons up to about 1000 BeV. Direct measurements have been made with artificially accelerated protons between 3 and 30 BeV. Above 10 BeV the K^+/π^+ production ratio is not strongly dependent upon the nature of the

* This work is supported in part through funds provided by the U. S. Atomic Energy Commission under Contract AT(30-1)2098.

[1] G. W. Clark and J. Hersil, Phys. Rev. **108**, 1538 (1957).
[2] H. V. Bradt, Ph.D. Thesis, Massachusetts Institute of Technology, 1961 (unpublished).
[3] H. V. Bradt and G. W. Clark, Bull. Am. Phys. Soc. **6**, 263 (1961). This reference should be corrected to read $P_u=P_s=0.22\pm0.03$.
[4] H. V. Bradt and G. W. Clark, in *Proceedings of the International Conference on Cosmic Rays and the Earth Storm at Kyoto, 1961*, published in J. Phys. Soc. Japan **17**, Suppl. A-III (1962), Vol. III, p. 300.
[5] S. Bennett and K. Greisen, Phys. Rev. **124**, 1982 (1961).
[6] P. J. Hayman and A. W. Wolfendale, Nature **195**, 166 (1962).
[7] J. E. Allen and A. J. Apostolakis, Proc. Roy. Soc. (London), **265A**, 117 (1961).
[8] W. Pak, S. Ozaki, B. P. Roe, and K. Greisen, Phys. Rev. **121**, 905 (1961).
[9] W. R. Sheldon and N. M. Duller, Nuovo Cimento **23**, 63 (1962).
[10] F. Ashton and A. W. Wolfendale, *Proceedings of the International Conference on Cosmic Rays and the Earth Storm at Kyoto, 1961*, published in J. Phys. Soc. Japan, **17** Suppl. A-III (1962), Vol. III, p. 313.
[11] L. F. Hansen and W. B. Fretter, Phys. Rev. **118**, 812 (1960).
[12] M. Koshiba and M. F. Kaplon, Phys. Rev. **97**, 193 (1955).
[13] B. Edwards, J. Losty, D. H. Perkins, K. Pinkau, and J. Reynolds, Phil. Mag. **3**, 237 (1958).
[14] A. G. Barkow, B. Chamany, D. M. Haskin, P. L. Jain, E. Lohrmann, M. W. Teucher, and M. Schein, Phys. Rev. **122**, 617 (1961).
[15] J. Duthie, P. H. Fowler, A. Kaddoura, D. H. Perkins, and K. Pinkau, Nuovo Cimento **24**, 122 (1962).

target or upon the incident-proton energy. However, it increases with both the momentum and the angle from the incident-proton beam of the secondary beam under analysis. Values between 0.08 and 0.27 are observed.[16–19]

Hayakawa[20] and others[21,22] have developed the theory relating the observed polarization of cosmic ray muons to the differential energy spectrum of the parent particles. If the parents are solely pions and if their spectrum obeys a power law with exponent α_π the theory predicts the muon polarization P_π indicated in Fig. 1. For example, if we assume a value $\alpha_\pi = 1.9$[23] for a sea-level experiment with small cover and allow for a 5% reduction of the polarization in the material traversed by the muon,[20] we expect a polarization of 0.25. In the case of $K_{\mu 2}$ decays the theory predicts, as indicated in Fig. 1, a muon polarization P_K which is near to unity for any power-law spectrum with exponent α_K greater than 1.5.

The theory shows that the polarizations P_π and P_K are dependent upon α_π and α_K respectively, and not upon the pion and kaon energy if, in fact, each spectrum obeys a power law. Thus a change of polarization with the production energy of the muons in the sample could be due to a change in the relative proportion of almost perfectly polarized muons of kaon parentage, and this, in turn, would reflect a variation in the K/π production ratio. [We define this ratio as the relative numbers of $K_{\mu 2}^+$ to π^+ mesons at production in the total (rest plus kinetic) energy interval dE at E]. One should note that depolarization in the absorber above the detector has been shown to be small and essentially independent of the thickness of the absorber[20] provided that there is little or no anomalous scattering of muons as they traverse nuclear matter, as recent experimental evidence indicates.[24–26]

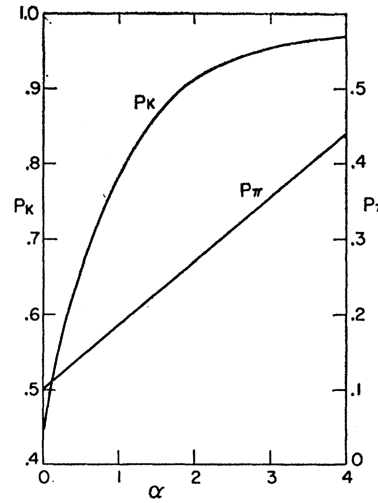

FIG. 1. Polarization, according to the theory of Hayakawa (see Ref. 20), of muons which derive from kaons P_K, or from pions P_π, as a function of the exponent α of an assumed kaon or pion power law differential energy spectrum. The curves do not take into account depolarization in the material traversed by the muons, a 5% effect.

The experiment of Hersil and Clark[1] yielded a polarization of 0.19 ± 0.06 which corresponds to $\alpha_\pi = 1.2 \pm 0.7$ if all muons derive from pions. Subsequent sea-level measurements at this depth (~ 200 g/cm^2 air equivalent)[21,27–31] confirmed this result.

Measurements of the polarization at depths greater than 200 g/cm^2 air equivalent have been made by several authors using delayed coincidence techniques[27,29,30,32,33] or cloud chambers.[31,34] There is some indication that the polarization does increase with depth[27,30] but in view of the large statistical errors these results are also consistent with a polarization which is constant with depth. Results with good statistical accuracy obtained at 200 g/cm^2 air equivalent[28] (0.21 ± 0.03) and at about 1000 g/cm^2 air equivalent[33] (0.25 ± 0.03) are apparently consistent with there being no increase with depth. However since there are certain systematic errors (e.g., electron scattering) which make it difficult to deduce the absolute polarization from the data, and since these errors may vary from one detector to another, one cannot compare these experimental results directly with one another.

[16] V. T. Cocconi, T. Fazzini, G. Fidecaro, M. Legros, N. H. Lipman, and A. W. Merrison, Phys. Rev. Letters **5**, 19 (1960).

[17] G. von Dardel, R. M. Mermod, G. Weber, and K. Winter, *Proceedings of the 1960 Annual International Conference on High-Energy Physics at Rochester* (Interscience Publishers, Inc., New York, 1960), p. 837.

[18] W. F. Baker, R. L. Cool, E. W. Jenkins, T. F. Kycia, S. J. Lindenbaum, W. A. Love, D. Lüers, J. A. Niederer, S. Ozaki, A. W. Read, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **7**, 101 (1961).

[19] V. L. Fitch, S. L. Meyer, and P. A. Piroué, Phys. Rev. **126**, 1849 (1962).

[20] S. Hayakawa, Phys. Rev. **108**, 1533 (1957). References 20 and 21 contain an error noted by Johnson (footnote 8 of Ref. 28). The results presented in Ref. 20 can be corrected as follows: Eq. (13) (valid for $\alpha \neq 0, \neq 1$) substitute $\alpha - 1$ for α; Eqs. (13b) and (13c) valid for $\alpha = 0$ and $\alpha = 1$, respectively; Eq. (15) substitute $\alpha + 1$ for α.

[21] J. M. Fowler, H. Primakoff, and R. D. Sard, Nuovo Cimento **9**, 1027 (1958).

[22] I. I. Gol'dman, Zh. Eksperim. i Teor. Fiz. **34**, 1017 (1958) [translation: Soviet Phys.—JETP **7**, 702 (1958)].

[23] M. Gardener, D. G. Jones, F. E. Taylor and A. W. Wolfendale, Proc. Phys. Soc. (London) **80**, 697 (1962).

[24] P. L. Connolly, J. G. McEwen, and J. Orear, Phys. Rev. Letters **6**, 554 (1961).

[25] G. E. Masek, L. D. Heggie, Y. B. Kim, and R. W. Williams, Phys. Rev. **122**, 937 (1961).

[26] M. Bardon, P. Franzini, and J. Lee, Phys. Rev. **126**, 1826 (1962).

[27] A. I. Alikhanyan, B. A. Dolgoshein, B. I. Luchkov, and V. I. Ushakov, in *Proceedings of the Moscow Cosmic Ray Conference, July 6–11 1959* (International Union of Pure and Applied Physics, Moscow, 1960), Vol. I, p. 318.

[28] C. S. Johnson, Phys. Rev. **122**, 1883 (1961).

[29] V. V. Barmin, V. P. Kanavets, and V. V. Morozov, Zh. Eksperim. i Teor. Fiz. **39**, 986 (1960) [translation: Soviet Phys.—JETP **12**, 683 (1961)].

[30] B. Dolgoshein, B. Luchkov, and V. Ushakov, Zh. Eksperim. i Teor. Fiz. **42**, 949 (1962) [translation: Soviet Phys.—JETP **15**, 654 (1962)].

[31] S. N. Sen Gupta and M. S. Sinha, Proc. Phys. Soc. (London) **79**, 1183 (1962).

[32] N. M. Kocharyan, Z. A. Kirakosyan, E. G. Sharoyan, and A. P. Pykalov, Zh. Eksperim. i Teor. Fiz. **38**, 18 (1960) [translation: Soviet Phys.—JETP **11**, 12 (1960)].

[33] A. I. Alikhanyan, T. L. Asatiani, E. M. Matevosyan, and R. O. Sharkhatunyan, Zh. Eksperim. i Teor. Fiz. **42**, 127 (1962) [translation: Soviet Phys.—JETP **15**, 90 (1962)].

[34] B. A. Dolgoshein and B. I. Luchkov, Zh. Eksperim. i Teor. Fiz. **36**, 640L (1959) [translation: Soviet Phys.—JETP **9**, 445L (1959)].

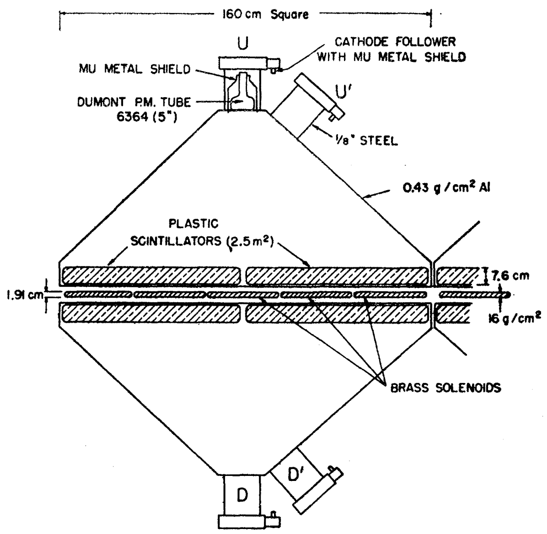

FIG. 2. One of the four pairs of detectors.

A determination of the absolute value of the K/π production ratio cannot be made from polarization measurements alone because of the uncertainty in α_K and α_π as well as the difficulty of deriving an absolute polarization from the data. On the other hand, one can obtain information about the possible *change* in the production ratio with increasing energy from measurements of the change in polarization with increasing depths of absorber. It turns out, however, that one can draw useful conclusions concerning such a change only if the polarization is measured with high statistical accuracy over a large range of depth as in the present experiment.

Even in the case of pure pion parentage the theoretical interpretation is complicated to a certain extent by the possibility that the muon polarization may vary with depth if the pion spectrum is not a power law. Account also must be taken of the competing probabilities for decay and interaction of the pions. Computations of muon polarizations which take these effects into consideration have been carried out by Berezinskii and Dolgoshein[35] and by Berezinskii.[36] Their results indicate that the polarization should increase from 0.19 at 15 g/cm² air equivalent to 0.33 at 500 g/cm² for muons arriving from the vertical, and that the polarization is greater for muons arriving at sea level with large zenith angles than for those arriving from the vertical; e.g., 0.25 at 60° and 0.19 at 0° at 15 g/cm². We shall include in our consideration of the K/π production ratios the possibility of such a depth and zenith angle variation of the polarization.

TABLE I. Summary of experimental conditions.

Run	Height above sea level	Cover, excluding atmosphere: Material	g/cm² (air equivalent)	Fig. no.
A	~150′	Plastic, Al	12±3	2, 3(b)
B	~30′	30 cm Pb[a]	190±10	2, 3(c)
C	~30′	38 cm Pb, 81 cm Fe[a]	720±25	2, 3(d)
D	~120′	14 m rock and clay[a] ($Z\approx10$, $A\approx20$)	3690±140	2, 3(e), 4

[a] Plus 12 g/cm² of trailer and detector materials.

[35] V. Berezinskii and B. Dolgoshein, Zh. Eksperim. i Teor. Fiz. 42, 1084 (1962) [translation: Soviet Phys.—JETP **15**, 749 (1962)].
[36] V. Berezinskii, Zh. Eksperim. i Teor. Fiz. 42, 485 (1962) [translation: Soviet Phys.—JETP **15**, 340 (1962)].

II. EXPERIMENTAL ARRANGEMENT

As in the earlier work[1] we measured the muon polarization by studying the effect of a magnetic field on the direction distribution of the decay electrons from muons that stopped in an absorber. The absorber was brass plates of 16 g/cm² thickness wound with an insulated copper wire solenoid. During alternate five minute periods a current in the solenoid provided a magnetic field of about 83 G. We measured the downward emitted decay electron flux with and without the magnetic field. Since the Larmor precession period of the muons at 83 G is small (0.89 μsec) compared to the mean life (2.2 μsec), an up-down asymmetry in electron directions was evidenced by a change in the counting rate during the application of the magnetic field. For the expected case of an initial asymmetry exhibiting a minimum in the decay probability in the downward direction, the effect of the magnetic field was to increase the DOWN counting rate. This method of observing only the downward flux avoids the complication

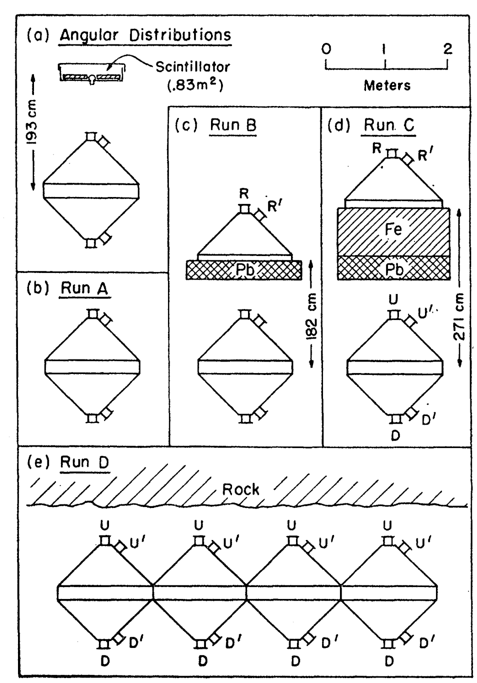

FIG. 3. Experimental configurations for runs A, B, C, and D and for the measurements of the zenith-angle distributions of muon arrival directions.

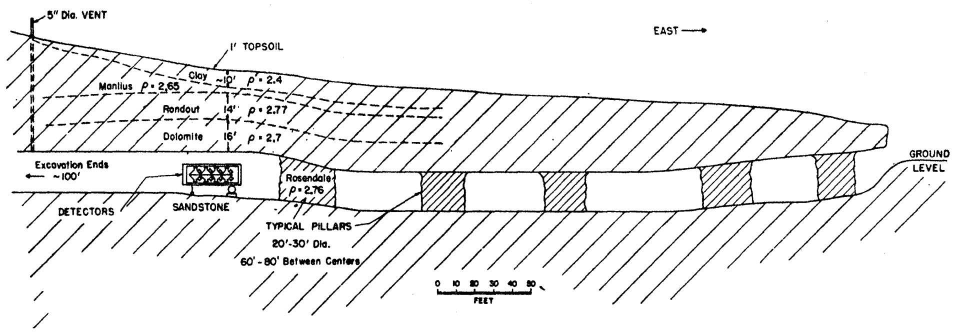

FIG. 4. Profile of experimental site at 3700 g/cm^2 cover. The measured densities in g/cm^3 of the rock are indicated by ρ. The character of the rock cover, well-known from contemporary mining operations in the immediate area, was obtained from J. Kelly (private communication).

of differences between up and down collection efficiencies involving electronic and geometrical effects.

For the absorber we chose brass because it has an atomic number that is, on the one hand, large enough so that it absorbs nearly all the negative muons before they decay and, on the other hand, small enough so that the decay electrons from positive muons are not too greatly scattered. The thickness of the absorber (16 g/cm^2) was chosen to be greater than the maximum effective range (12.3 g/cm^2)[37] of the most energetic decay electrons to prevent decay electrons from muons at rest in material above the absorber from reaching the DOWN detector and to facilitate an evaluation of the emerging electron flux.

Each of the eight detectors contained 2.4 square meters of plastic scintillator in a mosaic of four square pieces. Four of the detectors were above the brass plates and four were below. One pair is shown in Fig. 2. The scintillators were placed in $\frac{1}{16}$-in.-sheet aluminum cans painted white inside. Two Dumont 6364 5-in. photomultiplier tubes, each protected from magnetic fields by a mu metal shield and a steel cylinder, were used in each of the eight detectors. At each experimental site we carried out the basic calibration of the detectors for the purpose of setting the discriminator levels by measuring the distribution in size of pulses produced by single penetrating cosmic-ray particles selected with a coincidence telescope. In these measurements we found that the average sizes of pulses decreased by less than 30% from the center to the corner of the square scintillator mosaics. We performed routine checks of the calibration by measuring the integral distribution in size of pulses produced by uncollimated cosmic rays.

The directions of the currents in the coils wound on the twenty brass slabs were alternated so as to minimize the magnetic field at the photomultipliers. The resultant field without shields is estimated to have been about 0.025 G. The 15 kW of dc power required by the magnets was provided by an ac-dc motor generator set. The generator was rotated continuously and its field coils were alternately excited and de-excited to provide the ON-OFF cycling of the magnetic fields.

The entire apparatus was housed in three trailers so that it could be moved to various depths with a minimum of disturbance. The four detector pairs were placed side by side in a 28-ft aluminum "semitrailer." Measurements were made at four sites designated A, B, C, and D in order of increasing shielding over the detectors. The experimental conditions and configurations at these sites are summarized in Table I and Fig. 3. All four detector pairs were used to accumulate the data for run A, but only one detector pair was energized at a time in order to minimize events due to accidental coincidences. During this run, the equipment was located just outside the mine described below. Run D was made with the four pairs of detectors in parallel in a shallow mine. The results of a rough survey of this site are shown in Fig. 4. The site was well shielded in all directions, and the rock cover directly over the experimental apparatus was 3690±140 g/cm^2. Runs B and C were made in Cambridge, Massachusetts with an auxiliary detector in coincidence above the lead (and iron) absorber, as shown in Figs. 3(c) and 3(d). During runs B and C the upper detector was placed directly upon the absorber in order to obtain the maximum possible counting rates.

A delayed coincidence requirement selected the $\mu \rightarrow e$ decay events. During runs A and D the incoming muon was defined by the coincidence UU' and the anticoincidence requirement $\bar{D}$, where U', U, and D refer to the photomultipliers designated in Fig. 2, and $\bar{D}$ indicates an anticoincidence with the pulses from the D photomultiplier. After a passive 0.7-μsec delay, the $UU'\bar{D}$ requirement generated a 5.6-μsec gate (τ_1) during which decay electrons ($DD'\bar{U}$) could be detected. During runs B and C the incoming muon was further required to pass through detector R. In this case a coincidence between the phototubes R and R' (see Fig. 3) generated an 8.0-μsec gate (τ_2) which was placed in coincidence with the output of the delayed coincidence circuit just described. The discrimination levels of U', D', $\bar{U}$, $\bar{D}$, R, and R' were set quite low in order to detect all minimum ionizing particles. U was set to detect all stopping muons and D was set as low as was possible without incurring excessive accidental delayed coincidences (~8 MeV). The circuitry was completely transistorized with the exception of the scalers which were a standard tube design. Two photomultipliers were required in each detector to decrease events due to tube noise and to

[37] R. R. Wilson, Phys. Rev. 84, 100 (1951).

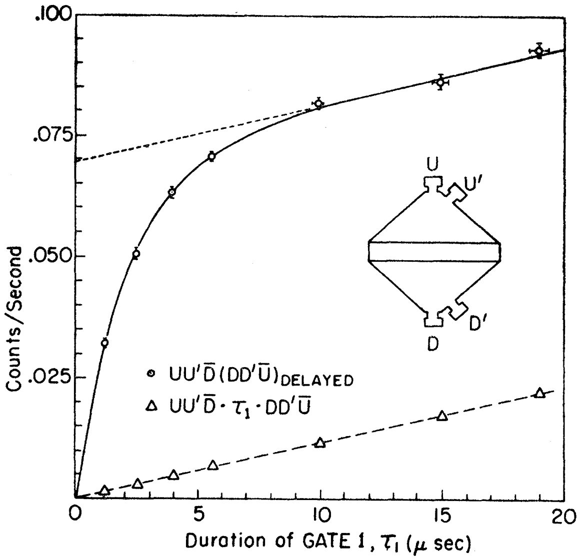

FIG. 5. Delayed coincidence rate, $UU'\bar{D}(DD'\bar{U})_{\text{delayed}}$ and the computed accidental delayed coincidence rate, $UU'\bar{D}\tau_1 DD'\bar{U}$, versus gate duration τ_1 at 3700 g/cm². The solid line represents the function $a\tau_1+b[1-\exp(-\tau_1/\tau)]$ with $\tau=2.2$ μsec, and with a and b adjusted to fit the data. The bar (e.g., $\bar{U}$ and $\bar{D}$) indicates anticoincidence.

eliminate spurious delayed coincidences due to "after pulsing."

The data were obtained from registers and clocks that were photographed once an hour. Since it was critically important that the sensitivity of the system not vary with the ON-OFF cycling, rates which should be independent of the magnetic field (e.g., UU', $DD'\bar{U}$, RR', etc.) were monitored by the camera. The camera also monitored rates (e.g., $UU'\bar{D}RR'$ and $DD'\bar{U}RR'$) which made available a continuous record of the accidental delayed coincidence rate.

III. PERFORMANCE OF THE EQUIPMENT

Table II summarizes the data from a typical experimental run. The first six data columns show the counting rates which should not have changed significantly when the magnetic field was turned on and off. As in this example the change in these rates was generally less than 0.1%, and almost always less than 0.2%. The last column shows the rates for the delayed coincidences which are affected by the magnetic field as expected, and which decrease by about 3% from ON to OFF.

Approximately 15% of the muon decay events were produced by muons which stopped in the upper one g/cm² of the DOWN scintillator or in adjacent materials and thus did not activate the anticoincidence. These decays were not affected by the magnetic field and caused a small systematic reduction in the difference between the ON and OFF rates which was accounted for in the analysis.

TABLE II. Comparison of counting rates with magnetic field ON and OFF (run D, 8 day average).

Magnetic field	Rates (sec⁻¹) UU' ±0.02	$\bar{D}$ ±0.13	Gate 1 ±0.009	DD' ±0.02	$\bar{U}$ ±0.12	$DD'\bar{U}$ ±0.010	Delayed coincidence ±0.00046
OFF	179.98	5815.4	27.563	196.50	4715.1	37.788	0.07008
ON	180.12	5807.3	27.577	196.54	4714.9	37.822	0.07253

We determined the relative proportion of muon decay events and accidental coincidences among the counts in the delayed coincidence channel in three ways. First we measured the rate of delayed coincidences as a function of the duration τ_1 of the delayed coincidence gate. The results obtained with all four detectors in parallel under 3700 g/cm² air equivalent of cover (site D) are shown in Fig. 5. The experimental points are fitted well by a function of the form $a\tau_1+b[1-\exp(-\tau_1/\tau)]$ with τ equal to the mean life of the muon (2.2 μsec), and with a and b adjusted to fit the data. The effect of the exponential muon decay is evident at small values of τ_1. At values of τ_1 large compared to τ the rate increases linearly with τ_1 as expected for accidental coincidences. Thus the term $a\tau_1$ represents the accidental coincidence rate. Secondly, we calculated the expected accidental coincidence rate from the gate durations τ_1 and τ_2 and the appropriate measured individual rates. The results for site D are shown as triangles in Fig. 5 where they may be compared with the linear term $a\tau_1$ obtained previously. The third way in which we determined the accidental rate was to reverse the logical roles of the UP and DOWN detectors by interchanging the cables. Muon decay events were thereby eliminated because stopping muons first had to traverse the UP counter which was then functioning in anticoincidence. Thus, the measured rate was purely due to accidental coincidences. It turned out that these three methods gave results in close agreement with one another. The second and third methods, which were capable of giving high statistical accuracy, were used for routine checks at all sites and always agreed within 5%. The uncertainty in the accidental rate introduced an uncertainty in the ratio of observed muon decay rates with magnetic field ON and OFF, $J_{\text{ON}}/J_{\text{OFF}}$, which was negligible in comparison with the statistical uncertainty.

The only significant correction that had to be made in comparing the results at various sites was for the zenith angle distributions of arrival directions of the observed muons. Measurements of the distributions and the resultant corrections are discussed later.

The statistical behavior of the data was checked by the χ^2 test and no evidence of unsteady behavior was found in any run.

IV. EXPERIMENTAL RESULTS

The running time and number of detected muons (after subtraction of accidentals) are presented in Table III for each run. Also the rates and ratios of rates necessary for computation of the ratios of polarizations are presented in this table. The accidental delayed coin-

TABLE III. Summary of experimental results.

Run	Running time (days)	Measured rates (sec^{-1}) ON	OFF	Accidental (±2%)	Number of muons[a]	J_{on}[b]/J_{off} $=\phi^{-1}$	Nominal polarization kP_i[c]	P_i/P_A[d]
A	18	0.18176	0.17639	0.0522	~195 000	1.0432±0.0055	0.23±0.03	...
B	46	0.047994	0.046560	0.00411[e]	~171 000	1.0335±0.0051	0.16±0.02	0.710±0.135
C	62	0.018751	0.017948	0.00178[e]	~89 000	1.0488±0.0074	0.23±0.03	0.988±0.187
D	71	0.072310	0.069884	0.00532	~410 000	1.0376±0.0034	0.21±0.02	0.929±0.139

[a] Approximate number after subtraction of accidentals.
[b] Ratio of rates with magnetic field ON and OFF after subtraction of accidentals.
[c] Nominal polarization kP_i comparable to polarization P quoted by other authors (see Sec. V).
[d] Ratio of polarization at site $i=B$, C, or D to polarization at site A (see Sec. V).
[e] Average of the ON and OFF values which differ by less than 0.7%.

cidence rates were computed from the gate durations τ_1 and τ_2 and from the time averages over an entire run of the appropriate observable rates, e.g., $UU'\bar{D}$, $DD'\bar{U}$, $UU'\bar{D}RR'$, etc. The drift of the various rates was sufficiently small so that this procedure did, in fact, give an accurate value of the time average accidental rate. For runs B and C the accidental rate was determined by adapting to this experiment the formulas derived by Schiff[38] for the accidental rate of a threefold coincidence.

V. INTERPRETATION

The calculation of Clark and Hersil[1] for the intensity of decay electrons emerging from a semi-infinite brass absorber has been extended to include the effect of the DOWN discriminator setting and the zenith angle of the muon arrival direction. As in the former calculation we used the energy and direction distribution for the decay electrons which is given by the theory of Lee and Yang.[39] We assumed a linear range-energy relation for the electrons in brass and neglected the scattering of the electrons in the brass. For muons with arrival directions at the zenith angle α and polarization P along their trajectories, the resulting expression for the flux of electrons emerging from the brass in the downward direction is

$$J(\alpha)\propto 1-k'\xi PS(u')b(\omega)\cos\alpha\,, \tag{1}$$

where $S(u')$ is a slowly varying function of the discriminator setting u' and approximately equal to 0.30, ξ is the asymmetry parameter in the Lee-Yang theory which has been found to be unity,[40] and $b(\omega)$ is a function of the precessional frequency of the muons such that $P'=b(\omega)P$ represents the apparent polarization during the precession. A calculation similar to that of the previous work,[1] but which takes into consideration the finite duration of the gate (τ_1), yields for the parameters of this experiment and with the magnetic field ON the value

$$b_{\rm ON}=+0.0717_{-9\%}^{+1\%}.$$

With no magnetic field, $P'=P$ and $b_{\rm OFF}=1$. The constant k' takes into account several sytematic errors and includes among others the effect of: (1) A 5% depolarization of the muons as they traverse the absorber and the material above the absorber[20], (2) a 15% background of muon decay events which is due to muons that stop in the top one g/cm^2 of the lower scintillator or other material near this scintillator and thus are not affected by the magnetic field, and (3) the decay of negative muons which reduce the observed asymmetry by about 5%. Thus,

$$k'=(0.95)(0.85)(0.95)k\,, \tag{2}$$

where k includes other systematic effects which have not been evaluated, e.g., (4) the scattering of the decay electrons as they traverse the absorber, and (5) a small but possibly significant depolarization of the positive muons at rest in the absorber as reported by Johnson.[28] The polarization P may then be related to the observed ratio of fluxes $\phi=J_{\rm OFF}/J_{\rm ON}$ by the expression

$$P=\frac{1-\phi}{Sk'(1-b_{\rm ON})}\frac{1}{\langle\cos\alpha\rangle}, \tag{3}$$

where we have omitted terms in $b\xi PS\cos\alpha$ which are small compared to the statistical errors. The angular brackets indicate an average over all detected muons.

The zenith angle distributions of arrival directions of the detected muons $i(\alpha)$ were obtained from an auxiliary experiment. We assumed a distribution $i(\alpha)=i_v\cos^\nu\alpha$ and determined values of ν from a comparison of the delayed coincidence rates due to muons arriving in our detector from the entire upper hemisphere and those arriving from near the vertical as defined by a 1-meter diam scintillator placed above one of the four detectors [see Fig. 3(a)]. The rates were accumulated concur-

TABLE IV. Zenith angle distributions of muon arrival directions.

Run	ν[a]	$\langle\cos\alpha\rangle$
A	$4.4_{-0.6}^{+0.7}$	0.844±0.016
B	3.3±0.7	0.933±0.003
C	3.0±0.7	0.960±0.001
D	2.9±0.3	0.796±0.013

[a] The values of ν for runs A and D are based on measurements described herein, but for runs B and C are estimates only.

[38] L. I. Schiff, Phys. Rev. **50**, 88 (1936).
[39] T. D. Lee and C. N. Yang, Phys. Rev. **105**, 1671 (1957).
[40] R. J. Plano, Phys. Rev. **119**, 1400 (1960).

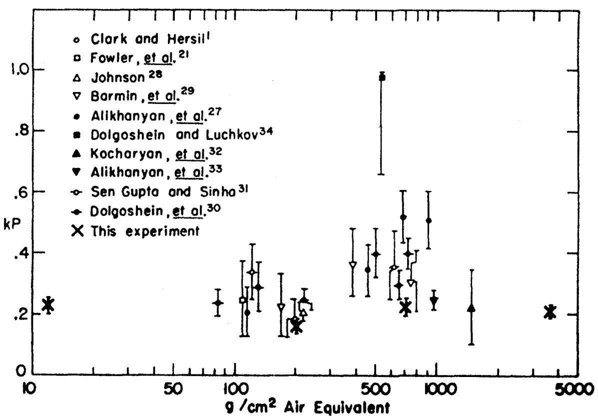

FIG. 6. Reported nominal polarizations kP of muons at rest versus material over the absorber, not including the atmosphere. The factor k takes into account systematic errors which are difficult to evaluate and which may vary from experiment to experiment.

rently and the solid angle factor of the telescope was computed numerically. The results for sites A and D are presented in Table IV. Due to the telescopic configuration for runs B and C, our final conclusions do not depend strongly upon the values of ν for these sites. Accordingly values which are consistent with the measured values at sites A and D were adopted and are presented in Table IV with appropriate limits of error. Values of $\langle\cos\alpha\rangle$ are also presented in Table IV. The solid angle factors of the telescopes used in the computation of $\langle\cos\alpha\rangle$ for runs B and C were also computed numerically. The measured values of ν are consistent with experimental and theoretical results of other authors.[7,10,41,42]

Values of kP computed from the experimental results (Tables III and IV) and Eqs. (2) and (3) are presented in Table III. The quantity kP is equivalent to the polarization P quoted by other authors in that it includes similar but not necessarily identical systematic errors. Therefore we present a summary of reported results in Fig. 6 as a function of shielding over the detectors. It appears that all the data are consistent with a constant nominal polarization kP equal to the average value $\langle kP\rangle_{av}=0.207\pm0.013$ from about 12 g/cm^2 to 3700 g/cm^2 air equivalent.

Ideally we would like to compare the experimental quantity kP with theoretical values of the polarization derived from various assumed production spectra of kaons and pions. However, the unknown systematic errors in the quantity k render this approach impossible. On the other hand, the ratio of the polarizations at two sites does not depend on k. If we label the sites i and A we have the relation

$$\frac{P_i}{P_A}=\frac{(1-\phi)_i}{(1-\phi)_A}\frac{\langle\cos\alpha\rangle_A}{\langle\cos\alpha\rangle_i} \quad (4)$$

The experimental ratio on the right-hand side of Eq. (4) is practically independent of the five systematic errors listed above (e.g., the scattering of electrons in the brass absorbers), fluctuations in the factors $S(u')$ and b_{ON}, and the effect of the magnetic field upon the trajectories of the muon and decay electron. Consideration of these effects indicates that in each case the ratio is affected by an amount which is small compared to the statistical errors. The values of this ratio computed from the data of the present experiment are presented in Table III.

We now proceed to evaluate the left-side of Eq. (4) in terms of several assumed kaon and pion spectra for comparison with the experimental results. The polarization P is obtained from an average of the muon polarization over the pion and kaon spectra as indicated in Eq. (5).

$$P=\frac{\int_{E_{\pi1}}^{E_{\pi2}}P_\pi(U,E)Q_\pi(U,E)dUf_\pi(E)\pi(E)dE+\int_{E_{K1}}^{E_{K2}}P_K(U,E)Q_K(U,E)dUf_K(E)K(E)dE}{\int_{E_{\pi1}}^{E_{\pi2}}Q_\pi(U,E)dUf_\pi(E)\pi(E)dE+\int_{E_{K1}}^{E_{K2}}Q_K(U,E)dUf_K(E)K(E)dE}, \quad (5)$$

$P_\pi(U,E)$ is the polarization of muons with total laboratory energy U at production which derive from π^+ mesons with total energy E; $\pi(E)dE$ the differential energy spectrum of positive pions at production; $f_\pi(E)$ the probability at a pion of energy E will survive interaction until decay; and $Q_\pi(U,E)dU$ the probability that a pion of energy E will decay to a muon of energy U in dU. Similar expressions are defined for the $K_{\mu2}^+\to\mu^++\nu$ decay.

Since the observed muons must stop in a thin absorber, their energy at production U is a function of their position of origin in the atmosphere. For the moment we assume that they are all created with a unique energy U near the top of the atmosphere. Thus we can adopt for the integration in Eq. (5) the following limits which are appropriate for the two-body decays at relativistic energies.

$$E_{K1}=E_{\pi1}=U \quad (6a)$$

$$E_{\pi2}=(m_\pi^2/m_\mu^2)U=1.74U \quad (6b)$$

$$E_{K2}=(m_K^2/m_\mu^2)U=21.8U \quad (6c)$$

[41] W. L. Kraushaar, Phys. Rev. **76**, 1045 (1949).

[42] E. P. George, in *Progress in Cosmic Ray Physics* (North-Holland Publishing Company, Amsterdam, 1952), Chap. VII.

From Hayakawa's results,[20] we obtain:

$$P_\pi(U,E)=\frac{UU_{\mu\pi}^*}{pp_{\mu\pi}^*}\frac{E}{m_\pi}\frac{m_\mu^2}{pp_{\mu\pi}^*}, \quad (7)$$

where p is the laboratory momentum of the muon and $U_{\mu\pi}^*$ and $p_{\mu\pi}^*$ represent the total energy and the momentum of the muon in the rest system of the parent pion. Equation (10) of Ref. 20 yields the probability Q_π.

$$Q_\pi(U,E)dU=m_\pi dU/2p_{\mu\pi}^*E \quad (8)$$

The survival probability f_π follows directly if we take the pion to be relativistic and the atmospheric density equal to x/H where x is the atmospheric depth of the pion and H is the characteristic length of the atmosphere (6.46×10^5 cm).[43] Thus

$$f_\pi(E)=E_{0\pi}/[E_{0\pi}+E(x/\lambda_\pi)], \quad (9)$$

where λ_π is the interaction mean free path of the pion, and $E_{0\pi}=m_\pi c^2 H(c\tau_0)^{-1}=117$ BeV, where τ_0 is the pion mean life at rest. In view of our assumption concerning height of production it is reasonable to take $x=\lambda_p=\lambda_\pi$ where λ_p is the mean free path of the primary protons. For kaons we take $E_{0K}=872$ BeV and $x=\lambda_p=\frac{1}{2}\lambda_K$.

We take the pion spectrum at production to be

$$\pi(E)dE=\frac{\pi_0}{(0.3+E)^{2.64}}dE, \quad (10)$$

where E is expressed in BeV. This expression is in approximate agreement from 1-6 BeV with the pion spectrum deduced by Olbert[44] and from 10-100 BeV with that of Pine *et al.*[45] We now assume that the ratio of the kaon spectrum to the pion spectrum is a power law of the form

$$K(E)/\pi(E)=a(0.3+E)^\delta. \quad (11)$$

Results obtained from accelerator experiments of Cocconi *et al*[16] and Baker *et al*[18] indicate that the production ratio $K_{\mu2}^+(E)/\pi^+(E)$ is about 0.10 at $E=1.4$ BeV. Thus the $K_{\mu2}^+$ differential energy spectrum we shall use in Eq. (5) is

$$K(E)dE=\frac{\pi_0 0.1dE}{1.7^\delta(0.3+E)^{2.64-\delta}}. \quad (12)$$

Equations (10) and (12) are plotted in Fig. 7, the latter for several values of δ. The object of our analysis is to place limits on the possible values of δ on the basis of the experimental data.

Equation (5) was evaluated and the ratios of polarizations were obtained for various δ for each experimental site with the aid of an electronic computer. The ratios are plotted in Figs. 8–10 versus δ for comparison with the experimental results which are also indicated on these graphs. In addition the absolute polarizations P from Eq. (5) are plotted versus δ for comparison with the experimental values of kP. The shape of these curves may be understood as follows. At a given energy the parameter δ determines the K/π production ratio according to Eq. (11). This ratio in conjunction with the accelerator data and Eq. (10) fixes the shape of the kaon-energy spectrum [Eq. (12)] which, with the pion-energy spectrum, determines the expected polarization.[46] In particular, as δ increases we note that: (1) The values of P increase as the fraction of high polarization muons in the observed sample becomes significant and then decrease as the slope of the kaon spectrum becomes nearly flat; (2) the increase is less pronounced in runs A and B than in C and D because the ratio is fixed at 0.10 at U_A(1.4 BeV) and at site B, for instance, a relatively small concentration of kaons is sufficient to imply a flat kaon-energy spectrum; and (3) the ratio of polarizations for $\delta\geq3.0$ depends critically upon the manner in which the individual polarizations go to zero, and therefore upon the assumed kaon and pion spectral shapes.

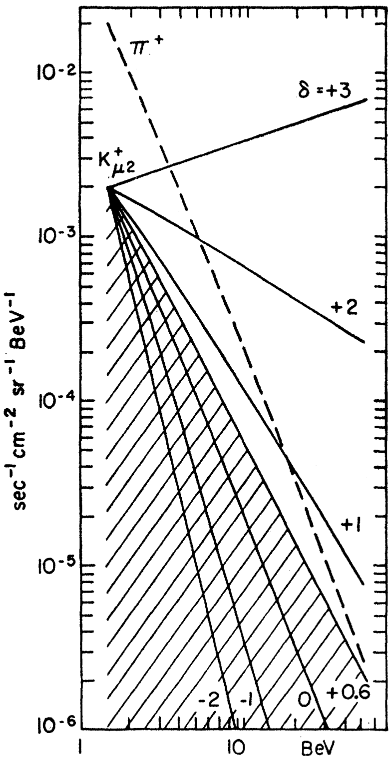

FIG. 7. Differential π^+ and $K_{\mu2}^+$ energy spectra, the latter for various δ, as a function of total (rest plus kinetic) energy. The spectra are constructed from Eqs. (10) and (12) where $\pi_0=0.156\times0.544=0.085$ is obtained from the pion spectrum of Pine *et al.* (see Ref. 45) and the positive excess of Bennett and Greisen (see Ref. 5). The experimental data are consistent with $K_{\mu2}^+$ spectra described by $-\infty<\delta<+0.6$ (hatched area).

It is clear from Fig. 10 that the experimental and theoretical ratios P_D/P_A are in agreement (within $2\frac{1}{2}$ standard deviations) for $-\infty<\delta<+0.6$ and $2.7<\delta<3.5$.

[43] P. H. Barrett, L. M. Bollinger, G. Cocconi, Y. Eisenberg, and K. Greisen, Rev. Mod. Phys. **24**, 133 (1952). Equation (9) is derived in a different context in this reference.
[44] S. Olbert, Phys. Rev. **96**, 1400 (1954).
[45] J. Pine, R. L. Davisson, and K. Greisen, Nuovo Cimento **14**, 1181 (1959).
[46] Previous computations of the K/π production ratios by the present authors (Ref. 4) and others (Refs. 30 and 31) did not take into account the effect of the production ratio upon the kaon spectrum.

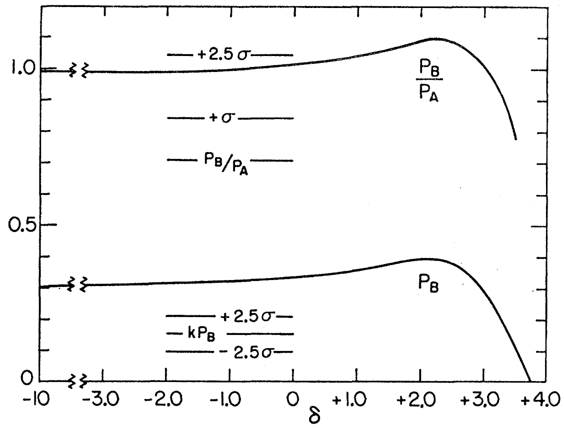

FIG. 8. Theoretical values of the ratio of polarizations P_B/P_A and the polarization P_B as a function of the parameter δ which specifies the $K_{\mu 2}{}^+$ spectrum (Fig. 7). The horizontal lines indicate the values obtained experimentally with limits of error based upon the standard deviation σ.

We reject the latter range of values since they predict an energy dependence and magnitudes for the K^+/π^+ production ratio which are at variance with accelerator data,[18] the zenith angle dependence of the muon spectrum,[8] and emulsion data.[14] Thus

$$-\infty < \delta < +0.6 \tag{13}$$

are the permissible values for δ. The corresponding kaon spectra are indicated by the cross-hatched area in Fig. 7. Finally, we note from Figs. 8 and 9 that the shape of the theoretical curves for sites B and C and the statistical accuracy of the results from these sites prevent us from drawing useful conclusions from them concerning production ratios. However, these results are consistent with Eq. (13).

Comparison of the theoretical polarizations P with the experimental values of kP in Figs. 8–10 indicates that systematic errors are important and that $k \approx 0.6$.

We obtained these results by assuming that: (1) The kaon spectrum varies smoothly with energy according to Eq. (12); (2) the muons observed at a given site are all produced at an atmospheric height defined by the production energy, U; (3) the polarization of the muons along their trajectories does not vary with the zenith angle of the flight path; and (4) the effect of the $K_{\pi 2}$, $K_{\pi 3}$, and $K_{\mu 3}$ decay modes is negligible. The first of these assumptions is necessary to our interpretation due to the dependence of the observed experimental effect upon the spectral shapes as well as the relative abundances of kaons and pions. The choice of an approximate power-law spectrum, Eq. (12), is reasonable in view of the similar character of the muon, pion, and primary energy spectra. Also, we have found that Eq. (13) is only moderately sensitive to large changes in the shape of the assumed kaon and pion spectra, e.g., if in Eqs. (10)–(12) 0.5 or 0.1 replaces 0.3, if 0.05 or 0.15 replaces 0.10, or if 2.0 or 4.0 replaces 2.64. For these cases the upper limit presented in Eq. (13) is at most 0.9. Since this limit is obtained by assuming that a fluctuation of $2\frac{1}{2}$ standard deviations occurred in the raw data, it is not necessary to revise Eq. (13).

It has been noted by Johnson[28] and Berezinskii and Dolgoshein[35] that the interpretation of polarization measurements is complicated by the variable heights of muon production. Muons which are produced low in the atmosphere have a correspondingly low energy if they are to stop in our absorber. In turn they are the decay products of low-energy parent mesons. Since the energy spectrum of pions is flatter at low energies than at higher energies,[23] the expected polarization is reduced. This effect is most pronounced for run A, i.e., 30% are produced in the lower half of the atmosphere and about 50% below an atmospheric depth of 300 g/cm^2.[44] We have used the latter height as the point of production for run A in order to obtain integration limits in Eq. (5). The mean height is somewhat lower. If we had chosen this lower height, the theoretical polarization P_A would be lowered, P_D/P_A would be increased, and δ would be lowered (see Fig. 10). Thus, Eq. (13) need not be changed.[47] The alternate choice of run B for the reference point raises the same difficulty to a somewhat lesser degree. This choice yields permissible values of δ which extend beyond, but which are consistent with Eq. (13).

The polarization of a muon with respect to its trajectory is not necessarily independent of the zenith angle of the trajectory. Muons in the observed sample which arrive at large zenith angles must traverse large thicknesses of atmosphere, rock etc. and must therefore be

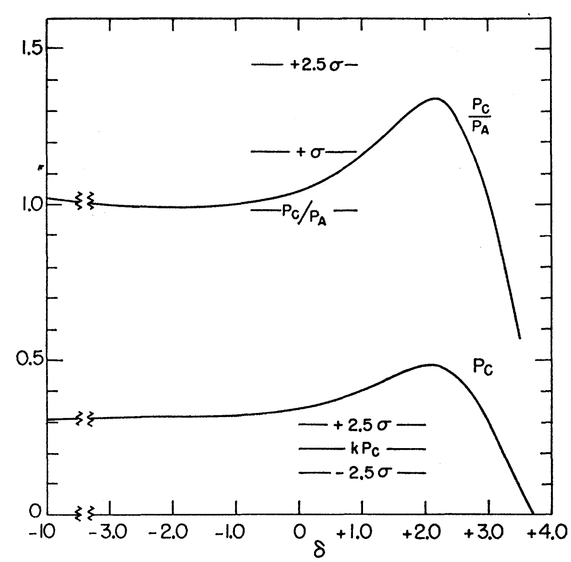

FIG. 9. Theoretical and experimental values of P_C/P_A and P_C. See caption of Fig. 8.

[47] The same argument applies if, as indicated in Ref. 23, the true pion spectrum is flatter at low energies than the spectrum used in this analysis.

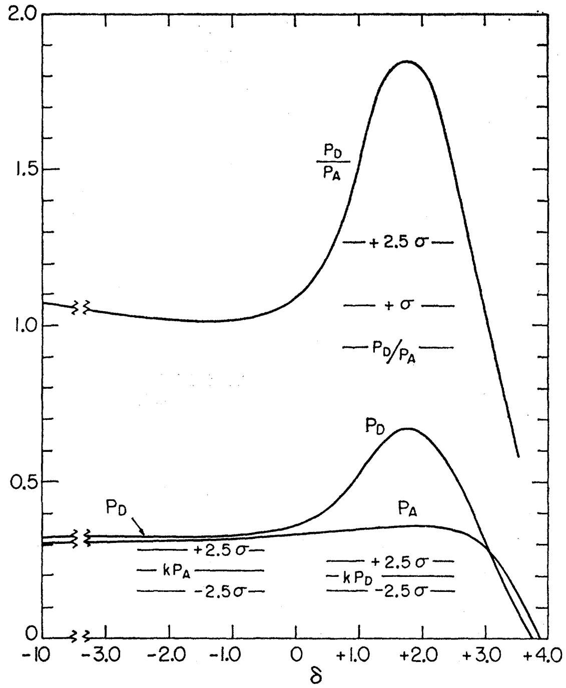

FIG. 10. Theoretical and experimental values of P_D/P_A, P_D and P_A. See caption of Fig. 8.

created with appreciably higher energies than those produced at small angles. At these energies, the relative abundances of the parent pions and kaons and the slopes of the spectra may result in a higher, or lower, polarization. We extended the numerical integration to include this effect and found that, despite a rather large change in polarization with zenith angle for certain values of δ (e.g., if $\delta=1.5$, $P_A=0.43$ at $\alpha=60°$ vice 0.35 at 0°, and $P_D=0.74$ at 60° vice 0.64 at 0°), the ratios of polarizations and consequently Eq. (13) are not strongly affected.

The effect of the $K_{\pi2}^+$, $K_{\pi3}^+$, and $K_{\mu3}^+$ decay modes upon the results quoted is found to be small due primarily to the relatively small probabilities for these modes and the fact that the observed muons derive on the average from higher energy and hence less numerous kaons than is the case for the $K_{\mu2}$ decay mode. However, Osborne[48] has pointed out that if K^+, K^0, and $\bar{K}^0$ mesons are produced in equal numbers, their numerous decay pions seriously dilute the muon polarization expected for a pure kaon source. Further dilution takes place if we assume that the $K_{\mu3}^+$ and $K_{\mu3}^0$ decay modes give rise to muons with zero polarization. We have computed the effect of these assumptions upon our results and find that they would increase the upper limit on δ from 0.6 to 1.3.

It is clear that our experimental polarization results do not exhibit the 70% increase (0.19 to 0.33) from 15 to 500 g/cm^2 predicted by Berezinskii and Dolgoshein[35] for a *pure pion* source.[49] We suggest that our experimental results and the predictions would be compatible if (1) the polarization measured in run A suffers from a statistical fluctuation and the true value is two standard deviations lower, and (2) the predicted increase is overestimated by a factor of two due to inaccuracy in the low-energy end of the assumed pion spectrum[44] and in the height distribution of muon production[50] used by the authors. The effect of the wide-opening angle of our detectors upon the results for run A, as suggested by Berezinskii and Dolgoshein[35] and Berezinskii,[51] is not important compared to the statistical errors because of the steep zenith angle distribution measured for this run (see Table IV).

The muon, pion, kaon, and primary energies appropriate for each run are summarized in Table V. We obtained the approximate average energy $\bar{E}_\pi$ of the π^+ mesons which contribute muons to each of the observed samples by weighting the energy with a power-law spectrum and the probability Q_π[Eq. (8)] and then integrating over the energy range indicated in Eqs. (6a) and (6b). The result is

$$\bar{E}_\pi=\frac{\alpha_\pi}{\alpha_\pi-1}\frac{1-(m_\mu^2/m_\pi^2)^{\alpha_\pi-1}}{1-(m_\mu^2/m_\pi^2)^{\alpha_\pi}}U. \qquad (14)$$

An expression similar to Eq. (14) is valid for the average kaon energy $\bar{E}_K$. The values of $\bar{E}_\pi$ and $\bar{E}_K$ given in Table V are for $\alpha_\pi=\alpha_K=2.6$.

The primary energies in Table V are obtained from the following considerations. Cloud chamber and emulsion measurements of multiplicities and inelasticities in high-energy collisions ($\geq 10^{10}$ eV) indicate that, on the average, a single secondary pion receives only a small fraction of the primary energy.[11,14] Furthermore, on the

TABLE V. Approximate total (rest and kinetic) energies of muons *at production*, their parents (pions and kaons), and the primaries which generate the parents.

	(BeV)							
	Muon	Pion (E_π)			Kayon (E_K)			Primary (U_p)
Run	(U)	Min	Av[a]	Max	Min	Av[b]	Max	Av[c]
A	1.4	1.4	1.8	2.4	1.4	2.4	31	16
B	1.8	1.8	2.3	3.1	1.8	3.0	39	21
C	3.0	3.0	3.8	5.2	3.0	4.9	65	34
D	10	10	12	17	10	16	220	108

[a] Computed from Eq. (14) for $\alpha_\pi=2.6$.
[b] Computed from an equation similar to Eq. (14) and $\alpha_K=2.6$.
[c] Computed from Eq. (17).

[48] J. Osborne, (private communication).

[49] Due to the simplifying assumption concerning height of muon production, our model predicts for a pure pion source only a small depth effect (see Fig. 10 for $\delta<-3$) and a zenith angle dependence approximately 1/5 that predicted by Berezinskii and Dolgoshein. Adoption of their model would not affect Eq. (13) (see text).

[50] M. Sands, Phys. Rev. **77**, 180 (1950).

[51] V. S. Berezinskii, Zh. Eksperim. i Teor. Fiz. **43**, 927 (1962) [translation: Soviet Phys.—JETP **16**, 657 (1963)].

basis of work by Vernov *et al.*,[52] Grigorov *et al.*,[53] and Grigorov,[54] Dormann[55] concludes that in a high-energy collision ($10^{10}-10^{12}$ eV) with a nucleus of an air atom, a primary cosmic ray gives, on the average, about 10% of its energy to a pion and, at the maximum, about 15%. Finally, we have made an empirical analysis of differential momentum spectra[18] of π^+ mesons produced at 9° from artificially accelerated proton beams of 10, 20, and 30 BeV which strike a beryllium target. It was found that these three spectra could be described approximately by the expression:

$$M(U_p,E)dEd\Omega \propto U_p^{1.8}\exp(-0.96E)dEd\Omega, \qquad (15)$$

where $M(U_P,E)$ is the number of pions projected with total energy E per unit energy interval into unit solid angle at 9° by a proton of total energy U_p, and where E and U_p are expressed in units of BeV. The intensity of primary cosmic-ray particles in the energy interval dU_p at U_p may be represented approximately by the formula

$$P(U_p)dU_p \propto U_p^{-\epsilon}dU_p, \qquad (16)$$

with $\epsilon=2.3$.[56] The product of Eqs. (15) and (16) gives the total number of pions produced in $d\Omega$ at 9° in dE at E from primaries in dU_p at U_p. This product was then used as the weighting factor to obtain the average energy $\bar{U}_p$ of primaries giving rise to pions of energy E at 9°. Averaging over primary energies between $1.5\bar{E}$ and $20\bar{E}$ we found the result

$$\bar{U}_p \approx 9\bar{E}. \qquad (17)$$

If we assume that Eq. (15) is valid for angles other than 9° and primary energies greater than 30 BeV, Eq. (17) may be adopted for our experimental conditions, and, furthermore, it is consistent with the work quoted above.

Thus, the muons recorded at the greatest depth (3700 g/cm^2) came from pions that had energies at production near 12 BeV, and these pions, in turn, came from interactions of primaries near 108 BeV.

ACKNOWLEDGMENTS

The authors are grateful to Robert Mudge, Charles Reed, James Nottingham, and William Smith whose technical assistance and advice at the experimental sites proved invaluable. The extensive film scanning and computations were ably performed by Allen Krieger and Bonnard Teegarden. Time on the IBM 7090 computer was provided by the Massachusetts Institute of Technology Computation Center and the program was prepared by Miss Elizabeth Campbell and Miss Elaine Miller. Permission to use the underground mine in Rosendale, New York was kindly granted by A. J. Snyder of the Century Cement Company and Hermann Knaust of Knaust Brothers, Inc. Finally, the authors would like to thank the Staff of the Laboratory for Nuclear Science for their support throughout the course of the experiment.

---

[52] S. N. Vernov, N. L. Grigorov, G. T. Zatsepin, and A. Ye. Chudakov, Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 493 (1955).
[53] N. L. Grigorov and V. S. Murzin, Izv. Akad. Nauk SSSR, Ser. Fiz. **17**, 21 (1953).
[54] N. L. Grigorov Dissertation, FIAN, 1955; Phys. Inst. Acad. Sci. USSR. (quoted in Ref. 55).
[55] L. I. Dorman, *Cosmic Ray Variations* (State Publishing House for Technical and Theoretical Literature, Moscow, 1957) [translation: Technical Documents Liaison Office, Wright-Patterson Air Force Base, Ohio].
[56] B. Peters, *Progress in Cosmic Ray Physics* (North-Holland Publishing Company, Amsterdam, 1952), Chap. IV.